A Spreadsheet Simulation Model of Sam Loyd's Carnival Dice Game

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Abstract

Everyone knows the expression "The house always wins". Expert gamblers, individuals with doctorates in statistics as well as those with little or no knowledge of probability understand that it unwise to bet against the house. The purpose of this paper is to illustrate how a classic gambling problem can be simulated to illustrate that probabilities can be used to accurately predict the results of a game of probability.

Keywords: simulation, model, probability, Sam Loyd

INTRODUCTION

The gambling and sports industry are big users of probability. These industries rely heavily on the use of probabilities to predict winning combinations and numbers of winners. Poker players seek to determine the odds of any particular hand winning over any other hand. Probabilities are used to determine the likelihood of a slot machine paying out. Numerous stats are calculated for just about every sport and for every game.

Everyone knows the expression “The house always wins”. Expert gamblers, individuals with doctorates in statistics as well as those with little or no knowledge of probability understand that it unwise to bet against the house. However, the statement does deserve some further analysis. After all, consider just how big the gambling industries around the world are and it becomes apparent that people believe that they can “beat the house.” Of course, what is essential to analyzing the player versus the house is the time line. Over the long term the house will always win. But in the short term, from one to two even five plays is it possible for the player, not the house to win?

PURPOSE AND OBJECTIVE

Helping students understand probability is a significant challenge in teaching statistics and math. Often the answers to probability questions appear to be counter-intuitive. Simulation can be a valuable tool to assist educators in this challenge. The purpose of this paper is to illustrate how a classic gambling problem, involving “Sam Loyd’s Carnival Dice Game,” can be simulated to illustrate that probabilities can be used to accurately predict the results of a game of probability. In this paper, we use this particular game to demonstrate how simulation can be a valuable tool for teaching various concepts of probability. However, before undertaking the simulation, we first provide a brief review of the extensive literature relating to the teaching of probability concepts.

LITERATURE REVIEW

The usefulness of statistics and probability for daily life, as well as the need for understanding basic stochastic processes, has been demonstrated to be absolutely essential in numerous professions. The necessity to include probability and statistics in the various teaching curricula has been repeatedly highlighted over the years (by Holmes 1980; Hawkins, et al., 1991; Vere-Jones 1995; Wild and Pfankuch 1999, Gal 2002).

In primary and secondary school levels, teachers frequently lack specific preparation in statistics education. As shown in Ortiz (1999) and Ortiz, et al. (2002), textbooks and curriculum documents prepared for primary and secondary teachers sometimes present a too narrow view of probability, and applications are at other times restricted to games of chance.

The disparity between intuition and conceptual development in probability and statistics has often been linked to a large number of paradoxes (Borovcnik, et al. 1991). The fact that professional statisticians still debate about different methodologies of inference (Fine 1971) demonstrates the comparative difficulty of trying to reconcile such disparities.

Borovcnik and Peard (1996) remark that counterintuitive results in probability are found even at very elementary levels. For example, the fact that having obtained a run of four consecutive heads when tossing a coin does not affect the probability that the following coin will result in heads is counterintuitive. These authors also suggest that probabilistic reasoning is different from logical reasoning because in a logical reasoning a proposition is always true or false and we have no complete certitude about a proposition concerning a random event.

In arithmetic an elementary operation can be reversed and can be represented with concrete examples. For young students, who are dependent on concrete situations for mathematical understanding, experiences with concrete examples are very important for abstracting the mathematical structure behind them. However, in the case of random experiment we obtain different results each time the experiment is carried out, meaning the experiment cannot be reversed. Even though simulation or experimentation with random processes have a very important function in children's intuition, they do not provide the key to how and why various problems are solved. It is only with the help of combinatorial schemes that students typically start to understand the solution of probabilistic problems.

Another reason for this difficulty is that stochastic processes live more in the area of mathematical applications than pure mathematics. In summary, stochastic processes are difficult to teach. To help students develop correct intuitions in this field, such as randomness or causality, it is important that they understand how various games can be solved with simulation techniques.

INTRODUCING THE GAME

Sam Loyd (1841-1911) is one of the best known writers in the field of recreational mathematics. In addition to the thousands of puzzles he authored in his lifetime, he was also well-known as a chess aficionado (Puzzles, Sam Loyd Puzzles, n.d.). The puzzle selected for this simulation model was originally printed in the privately published *Cyclopedia of Puzzles* (Loyd, 1914). The puzzle also appears later in *Mathematical Puzzles of Sam Loyd* (Gardner, 1959). The puzzle is re-printed below:

"The following dice name is very popular at fairs and carnivals, but since two persons seldom agree on the chances of a player winning, I offer it as an elementary problem in the theory of probability.

On the counter are six squares marked 1, 2, 3, 4, 5, 6. Players are invited to place as much money as they wish on any one square. Three dice are then thrown. If your number appears on one die only, you get your money back plus the same amount. If two dice show your number, you get your money back plus twice the amount you placed on the square. If your number appears on all three dice, you get your money back plus three times the amount. Of course if the number is not on any of the dice, the operator gets your money.

To make this clearer with an example, suppose that you bet 1 dollar on No. 6. If one die shows a 6, you get your dollar back plus another dollar. If two dice show 6, you get back your dollar plus two dollars. If three dice show 6, you get your dollar back plus three dollars.

A player might reason: the chance of my number showing on one die is 1/6, but since there are three dice, the chances must be 3 / 6 or 1/2, therefore the game is a fair one Of course this is the way the operator of the game wants everyone to reason, for it is quite fallacious.

Is the game favorable to the operator or the player, and in either case, just how favorable is it?" (Loyd, 1914).

The game is actually a fairly simple example of how probability determines the results and how when people do not understand how probabilities function they can make poor decisions. This of course is exactly what Loyd (1914) is referring to when he alludes to the fact that a player might believe that the game is fair and that the "house" or operator is counting on the players not understanding the odds.

METHODOLOGY OF CARNIVAL DICE GAME

The following is a statistical analysis of the expected result from the dice game. When one considers all of the possibilities for rolling 3 fair die, there are 216 possible outcomes. Suppose that the player places one dollar on the number 1, will he win or lose? Table 1 presents the 216 outcomes. The columns labeled D1, D2 and D3 represent the result of the three dice. The column labeled $ represents the resulting payoff if the player bet on die one. Note that the player wins $3 only if he gets a 1 on each of the 3 die, which only occurs once out of the 216 simulated rolls. Likewise, double ones, which result in a winning of $2, only occurs 15 times.

Table 1

Results of Loyd's Carnival Dice Game

D1	D2	D3	$	D1	D2	D3	$	D1	D2	D3	$	D1	D2	D3	$	D1	D2	D3	$	D1	D2	D3	$
1	1	1	3	2	1	1	2	3	1	1	2	4	1	1	2	5	1	1	2	6	1	1	2
1	1	2	2	2	1	2	1	3	1	2	1	4	1	2	1	5	1	2	1	6	1	2	1
1	1	3	2	2	1	3	1	3	1	3	1	4	1	3	1	5	1	3	1	6	1	3	1
1	1	4	2	2	1	4	1	3	1	4	1	4	1	4	1	5	1	4	1	6	1	4	1
1	1	5	2	2	1	5	1	3	1	5	1	4	1	5	1	5	1	5	1	6	1	5	1
1	1	6	2	2	1	6	1	3	1	6	1	4	1	6	1	5	1	6	1	6	1	6	1
1	2	1	2	2	2	1	1	3	2	1	1	4	2	1	1	5	2	1	1	6	2	1	1
1	2	2	1	2	2	2	-1	3	2	2	-1	4	2	2	-1	5	2	2	-1	6	2	2	-1
1	2	3	1	2	2	3	-1	3	2	3	-1	4	2	3	-1	5	2	3	-1	6	2	3	-1
1	2	4	1	2	2	4	-1	3	2	4	-1	4	2	4	-1	5	2	4	-1	6	2	4	-1
1	2	5	1	2	2	5	-1	3	2	5	-1	4	2	5	-1	5	2	5	-1	6	2	5	-1
1	2	6	1	2	2	6	-1	3	2	6	-1	4	2	6	-1	5	2	6	-1	6	2	6	-1
1	3	1	2	2	3	1	1	3	3	1	1	4	3	1	1	5	3	1	1	6	3	1	1
1	3	2	1	2	3	2	-1	3	3	2	-1	4	3	2	-1	5	3	2	-1	6	3	2	-1
1	3	3	1	2	3	3	-1	3	3	3	-1	4	3	3	-1	5	3	3	-1	6	3	3	-1
1	3	4	1	2	3	4	-1	3	3	4	-1	4	3	4	-1	5	3	4	-1	6	3	4	-1
1	3	5	1	2	3	5	-1	3	3	5	-1	4	3	5	-1	5	3	5	-1	6	3	5	-1
1	3	6	1	2	3	6	-1	3	3	6	-1	4	3	6	-1	5	3	6	-1	6	3	6	-1
1	4	1	2	2	4	1	1	3	4	1	1	4	4	1	1	5	4	1	1	6	4	1	1
1	4	2	1	2	4	2	-1	3	4	2	-1	4	4	2	-1	5	4	2	-1	6	4	2	-1
1	4	3	1	2	4	3	-1	3	4	3	-1	4	4	3	-1	5	4	3	-1	6	4	3	-1
1	4	4	1	2	4	4	-1	3	4	4	-1	4	4	4	-1	5	4	4	-1	6	4	4	-1
1	4	5	1	2	4	5	-1	3	4	5	-1	4	4	5	-1	5	4	5	-1	6	4	5	-1
1	4	6	1	2	4	6	-1	3	4	6	-1	4	4	6	-1	5	4	6	-1	6	4	6	-1
1	5	1	2	2	5	1	1	3	5	1	1	4	5	1	1	5	5	1	1	6	5	1	1
1	5	2	1	2	5	2	-1	3	5	2	-1	4	5	2	-1	5	5	2	-1	6	5	2	-1
1	5	3	1	2	5	3	-1	3	5	3	-1	4	5	3	-1	5	5	3	-1	6	5	3	-1
1	5	4	1	2	5	4	-1	3	5	4	-1	4	5	4	-1	5	5	4	-1	6	5	4	-1
1	5	5	1	2	5	5	-1	3	5	5	-1	4	5	5	-1	5	5	5	-1	6	5	5	-1
1	5	6	1	2	5	6	-1	3	5	6	-1	4	5	6	-1	5	5	6	-1	6	5	6	-1
1	6	1	2	2	6	1	1	3	6	1	1	4	6	1	1	5	6	1	1	6	6	1	1
1	6	2	1	2	6	2	-1	3	6	2	-1	4	6	2	-1	5	6	2	-1	6	6	2	-1
1	6	3	1	2	6	3	-1	3	6	3	-1	4	6	3	-1	5	6	3	-1	6	6	3	-1
1	6	4	1	2	6	4	-1	3	6	4	-1	4	6	4	-1	5	6	4	-1	6	6	4	-1
1	6	5	1	2	6	5	-1	3	6	5	-1	4	6	5	-1	5	6	5	-1	6	6	5	-1
1	6	6	1	2	6	6	-1	3	6	6	-1	4	6	6	-1	5	6	6	-1	6	6	6	-1

The number of $1 wins is 75. The number of $1 losses, indicated by a -1, is 125. Thus the probability of winning a game is 91/216 (.42) and losing is (.58). The expected value for the player is to lose 7.87 cents. The probabilities and expected payoffs are summarized in Table 2.

Table 2

Summary of Probabilities and Expected Values Loyd's Carnival Dice Game

Player Payoff	Probability	Expected Value
3	1/216	0.0139
2	15/216	0.1389
1	75/216	0.3472
-1	125/216	-0.5787
	Mean	-0.0787

SIMULATION RESULTS OF THE CARNIVAL DICE GAME

A simulation of the dice game is presented next. The dice game is good example of how the use of probabilities in spreadsheet simulation can verify the statistical analysis presented above. Table 3 displays a partial result of 100 simulated games. A description of each of the column values is shown below:

Column	Description
A	Row #
B	Roll #
C	Bet on #
D, E, F	Roll Result Die1, Die 2, Die 3
G, H, I	Match or No Match Bet Compared to Die 1, 2, 3
J	Number of Matches
K	Game Payoff
L	Running Game Payoff
M	# of Losses
N	# of Wins
O	# of Losses
P	# of $1 Wins
Q	# of $2 Wins
R	# of $3 Wins

Table 3

Simulation of Loyd's Carnival Dice Game

1\A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
2	#	Bet	D1	D2	D3	M1	M2	M3	Sum	Pay$	Cum$	n=-1	n>=1	#-1	#1	#2	#3
3	1	3	6	1	1	0	0	0	0	-1	-1	1	0	1	0	0	0
4	2	1	3	5	2	0	0	0	0	-1	-2	2	0	2	0	0	0
5	3	1	6	1	4	0	1	0	1	1	-1	2	1	2	1	0	0
6	4	6	6	5	6	1	0	1	2	2	1	2	2	2	1	1	0
7	5	2	5	3	5	0	0	0	0	-1	0	3	2	3	1	1	0
8	6	5	6	3	6	0	0	0	0	-1	-1	4	2	4	1	1	0
9	7	2	6	4	2	0	0	1	1	1	0	4	3	4	2	1	0
10	8	5	2	1	6	0	0	0	0	-1	-1	5	3	5	2	1	0
11	9	6	5	4	2	0	0	0	0	-1	-2	6	3	6	2	1	0
12	10	3	6	2	3	0	0	1	1	1	-1	6	4	6	3	1	0
13	11	1	1	6	4	1	0	0	1	1	0	6	5	6	4	1	0
14	12	2	3	3	4	0	0	0	0	-1	-1	7	5	7	4	1	0
15	13	3	6	2	4	0	0	0	0	-1	-2	8	5	8	4	1	0
16	14	5	3	5	2	0	1	0	1	1	-1	8	6	8	5	1	0
17	15	5	1	4	1	0	0	0	0	-1	-2	9	6	9	5	1	0
18	16	5	3	6	5	0	0	1	1	1	-1	9	7	9	6	1	0
19	17	6	2	1	6	0	0	1	1	1	0	9	8	9	7	1	0
20	18	3	1	6	6	0	0	0	0	-1	-1	10	8	10	7	1	0
97	95	6	4	5	2	0	0	0	0	-1	-9	54	41	54	37	4	0
98	96	2	3	2	1	0	1	0	1	1	-8	54	42	54	38	4	0
99	97	5	5	6	1	1	0	0	1	1	-7	54	43	54	39	4	0
100	98	6	4	2	6	0	0	1	1	1	-6	54	44	54	40	4	0
101	99	2	5	6	1	0	0	0	0	-1	-7	55	44	55	40	4	0
102	100	1	1	3	6	1	0	0	1	1	-6	55	45	55	41	4	0

Consider the last play. The player bets on 1. The rolls are 1, 3 and 6. Since there is one match the player wins $1. Cell L102 indicates the running payoff for this simulation is the player loses $6. Cells M102 and N102 indicate the house won 55% and the player won 45% of the simulated games. Cells O102-Q102 indicate house wins 55% of the games, 41% of the games the player wins $1, and 4% of the games the player matches 2 die with his bet and wins $2. Cell R102 indicates that a 3 die match does not occur during this simulation, thus the player does not experience a payoff of $3.

As discussed earlier, results can vary widely from simulation to simulation. Is it possible for the player to actually come out ahead and win over the course of 100 plays? Of course, the answer is yes. However based on the known probabilities it is not likely. Table 4 displays a summary of 100 simulated runs of n=100 games per simulation.

Table 4

Summary of 100 Simulations of Loyd's Carnival Dice Game (n=100)

	$ Payoff		Losses (%)	Wins %	Lose $1 (%)	Win $1 (%)	Win $2 (%)	Win $3 (%)
mean	-6.44		57.34	42.66	57.34	35.04	7	0.6122449
median	-9		58.5	41.5	58.5	34.5	7	0
max win	14	High	70	52	70	46	13	3
max loss	-30	Low	48	30	48	21	2	0
% wins	42							
% losses	58							

As expected the house wins 58% and the player wins 42%. The mean and median payoffs are -6.44 and -9. The best payoff for the player is winning $14. While the highest payoff for the house is $30. The mean and median percentages for the house winning are 57.3% and 58.5%. Correspondingly, the percentage mean and median wins for the house are 42.7 and 41.5 respectively. The highest and lowest percentage wins for the house are 70 and 48. For the player the high low percentages wins\losses are 52 and 30. The percentage of the simulations the player will win exactly $1 is approximately 35, with a high and low of 46 and 21. A player can expect to win $2 about 7% of the time. The high and low percentages for a $2 player win is 13 and 2. The probability of the player winning $3 is very low, with a mean of .61% and a median of probability of 0. Only 3 times in the course of 100 simulated games of 100 plays each, did the player win $3.

SUMMARY

Understanding probabilities and the counter-intuitive nature of related issues is often a challenge for educators and students. Games of chance can be valuable in helping educators to meet this challenge. Simulation models, such as the one discussed in this paper, can provide insight and serve as a teaching tool to helping individuals understand fluctuations in results from play to play. In this game of chance, Loyd's carnival dice game, the odds are clearly stacked in favor of the house. While, it proves possible that it is possible for the player to win some games, even a majority out of 100 plays, the reality is that in the long run the house will win.

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